Unifying Nucleon and Quark Dynamics at Finite Baryon Number Density

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Abstract

We present a model of baryonic matter which contains free constituent quarks in addition to bound constituent quarks in nucleons. In addition to the common linear σ-model we include the exchange of vector-mesons. The percentage of free quarks increases with baryon density but the nucleons resist a restoration of chiral symmetry.

1 Introduction

The possibility of creating high density baryonic matter has triggered several speculations about the appearance of a quark phase and/or diquark phase with or without chiral symmetry restoration. Most naive quark models with effective interactions have an inherent very low scale at which chiral symmetry is restored. This scale can be obtained from an effective potential calculation within a linear sigma model for example, and varies between 0.5 and 1.5 times normal nuclear density ρ_0. This density appears to be unrealistically low, because higher mass mesons (e.g. ω-mesons) are neglected in these models. The inclusion of higher mass mesons yields repulsive forces that drive the system away from the chiral symmetry restoration phase transition.

Relativistic Quantum Hadron Dynamics (QHD) has been a very successful mean field theory of nuclear matter for certain features of nuclei. It gives a description of nuclear saturation and of the spin-orbit potential in nuclei, which is close to that obtained phenomenologically. On the other hand, its compressibility $K_\infty > 500$ MeV is too high, compared to the empirical value $K_\infty = 210$ MeV [1]. Although due to confinement nucleon degrees of freedom replace quark degrees of freedom in vacuum, a growing part of the nucleon wave functions will overlap with increasing nuclear density, because of the finite size of the nucleons. By this, the substructure of the nucleon matters already before any phase transition, and so it is interesting to ask the question what chemistry of nuclear matter results if one combines both nucleons and quarks. How is baryonic matter built up when a fraction of baryonic charge can be delocalized in quarks, which have their own dynamics? In the seventies various schemes of covalent bonding were invented. The above idea goes far beyond these. It assumes the existence of an unconstrained quark substrate.

2 Quark nucleon model

We unify both quark and nucleon dynamics by a common linear σ-model Lagrangian via which both sectors communicate. In equilibrium we do not need complicated transition terms n $\leftrightarrow$ 3q.

The Lagrangian of our model reads

$$\mathcal{L} = \bar{\psi}_n(i\gamma^\mu D_\mu^{(vn)} - g_{sn}(\sigma + i\vec{\tau}\vec{\pi}\gamma_5))\psi_n + \bar{\psi}_q(i\gamma^\mu\partial_\mu - g_{sq}(\sigma + i\vec{\tau}\vec{\pi}\gamma_5))\psi_q$$
$$+ \frac{1}{2}\left((D_\mu^{(sv)}\vec{\pi})^2 + (D_\mu^{(sv)}\sigma)^2\right) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} - \mathcal{V}(\sigma^2 + \vec{\pi}^2),$$

with the covariant derivatives $D_\mu^{(vn/sv)} = \partial_\mu + ig_{vn/sv}\omega_\mu$, the field tensor $\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ and the spontaneous symmetry breaking potential $(m_s^2 < 0)$

$$\mathcal{V}(\sigma^2 + \vec{\pi}^2) = \frac{m_s^2}{2}(\sigma^2 + \vec{\pi}^2) + \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2.$$

The free constituent quarks with the same mass as quarks bound in nucleons represent larger subclusters of quarks, for which a definite baryonic center point can no longer be localized. These "free" quarks mimic the frequent switching of string configurations. In this picture, the ω-repulsion is identified with the repulsion of the string junctions.

The hadronic part is a modification of the model presented by Walecka [2], with the explicit mass term of the vector meson replaced by a coupling to the scalar mesons. Thus in mean field theory, the ω also gains its mass from the σ-field. This leads to a scaling of all hadron masses as suggested by Brown and Rho [3], who have also investigated the compatibility of nucleon and quark degrees of freedom at finite temperature and density [4]. As a consequence, the chiral symmetric phase with $\langle\sigma\rangle \approx 0$, i.e. very small ω-mass, is suppressed, because in the mean field approximation the ω-repulsion becomes very large.

We choose the following couplings: $g_{sq} = 3.23$, $g_{sn} = 3 \cdot g_{sq} = 9.69$, $m_s^2 = -0.26$, $\lambda = 30.0$, $g_{sv} = 8.41$, $g_{vn} = 9.5$. The couplings of the mesonic potential m_s^2 and λ are calculated from a renormalization group (RG) flow-equation approach for the quark part of the model [5]. The couplings yield a σ-mass of $m_\sigma = 720$ MeV and a nucleon mass of $m_n = 901$ MeV. The nucleon mass is less than 938 MeV because an explicit symmetry breaking term is not included in our Lagrangian. In the mean field approximation with constant expectation values $\bar{\sigma}$ and $\bar{\omega}$, the Lagrangian takes the form

$$\mathcal{L}_{MF} = \bar{\psi}_n(i\gamma^\mu\partial_\mu - g_{vn}\gamma^0\bar{\omega} - g_{sn}\bar{\sigma})\psi_n + \bar{\psi}_q(i\gamma^\mu\partial_\mu - g_{sq}\bar{\sigma})\psi_q$$
$$+ \frac{1}{2}g_{sv}^2\bar{\sigma}^2\bar{\omega}^2 - \frac{m_s^2}{2}\bar{\sigma}^2 - \frac{\lambda}{4}\bar{\sigma}^4.$$

3

The number of internal degrees of freedom is $\gamma_n = 4$ for the nucleons and $\gamma_q = 12$ for the quarks, because of 2 spin and 2 isospin states for both and in addition 3 different colors for the quarks. In the ground state, the nucleons and quarks build two Fermi gases with Fermi momenta k_{Fn} and k_{Fq}. They are connected to the conserved overall baryon density $\rho_B = \psi_n^\dagger \psi_n + \frac{1}{3}\psi_q^\dagger \psi_q$ through

$$(1-x)\rho_B = \frac{\gamma_n}{6\pi^2}k_{Fn}^3 \quad \text{and} \quad 3x\rho_B = \frac{\gamma_q}{6\pi^2}k_{Fq}^3 \, ,$$

with x denoting the concentration of quarks in the system.

Now one can calculate the energy density, which is[1]

$$\epsilon = \frac{\gamma_n}{8\pi^2}\left(\sqrt{k_{Fn}^2 + g_{sn}^2\bar{\sigma}^2}\left(k_{Fn}^3 + \frac{g_{sn}^2\bar{\sigma}^2 k_{Fn}}{2}\right) - \frac{g_{sn}^4\bar{\sigma}^4}{2}\log\left(\frac{k_{Fn} + \sqrt{k_{Fn}^2 + g_{sn}^2\bar{\sigma}^2}}{g_{sn}\bar{\sigma}}\right)\right)$$
$$+ \frac{\gamma_q}{8\pi^2}\left(k_{Fn} \leftrightarrow k_{Fq}, g_{sn} \leftrightarrow g_{sq}\right) + \frac{\gamma_n^2 g_{vn}^2}{72\pi^4 g_{sv}^2\bar{\sigma}^2}k_{Fn}^6 + \frac{m_s^2}{2}\bar{\sigma}^2 + \frac{\lambda}{4}\bar{\sigma}^4 \, ,$$

where the mean field value $\bar{\sigma}$ in the ground state is self-consistently given by minimizing ϵ with respect to $\bar{\sigma}$.

3 Results

The result of our calculation is plotted in Fig.1. To the left the $x = 0$ boundary curve shows the pure hadronic phase. The right boundary at $x = 1$ shows the same for pure quark matter with a kink due to the first order chiral phase transition. This transition persists only for a quark concentration $x \gtrsim 0.99$, where at the endpoint the phase transition is of second order. The white line denotes the stable configuration with minimal energy at each density and shows a smooth transition between a vanishing quark concentration in vacuum and an asymptotic quark concentration $x_\infty \approx 0.9$ as $\rho_B \to \infty$.

This curve of the energy minimum, which represents the equation of state (EOS) of nuclear matter, is also plotted as a 2D projection in Fig.2 together with the advanced nuclear physics calculations by Pandharipande et. al. [6]. Whereas quark matter and nuclear matter alone do not bind, the mixture of both binds at a saturation density $\rho_S = 1.14\rho_0$ with $E_B = 16$ MeV.

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[1]The first term in the second line is the same as the nucleon term in the first line with interchanged momenta and couplings
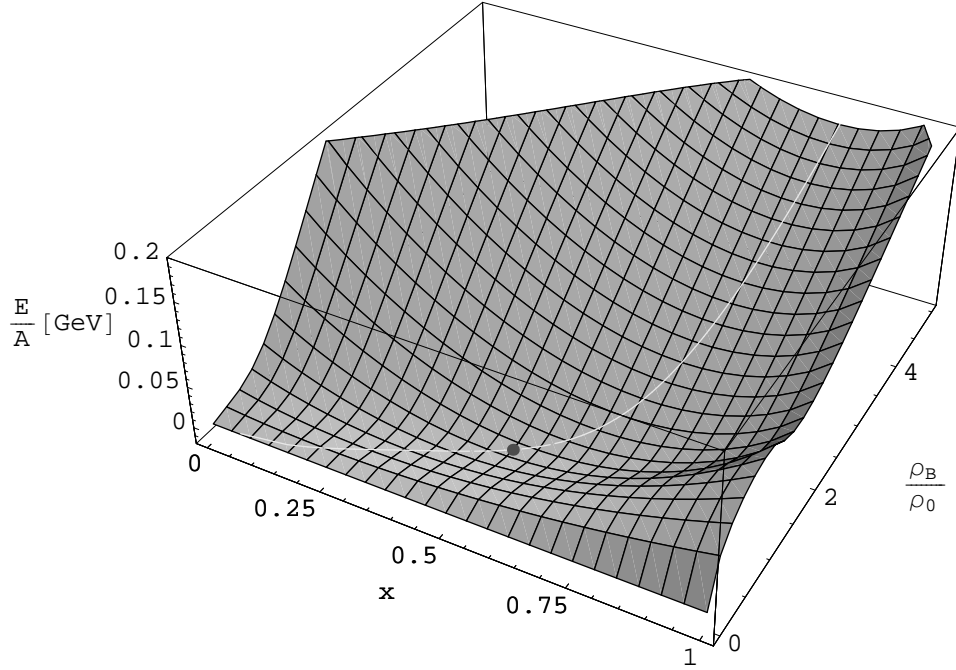
Figure 1: Binding energy per baryon $\frac{E}{A} = \frac{\epsilon(\rho_B) - \epsilon(0)}{\rho_B} - m_N$ as a function of the independent variables x and ρ_B given in units of normal nuclear matter density $\rho_0 = 0.17$ fm^{-3}

The Dirac mass of the nucleon at saturation is $m_D^* = g_{sn}\bar{\sigma}_s = 581$ MeV. As pointed out in [7] this quantity is not the relativistic analogon of the nonrelativistic effective mass measured in nuclear experiments. The corresponding quantity is the relativistically equivalent effective mass m_e^*. We get $\frac{m_e^*}{m_n} = 1 - \frac{g_{vn}\bar{\omega}}{m_n} = 0.71$ compared to an experimental value of $\frac{m^*}{m_n} \approx 0.74$ [7]. As in the standard Walecka model the compression modulus $K_\infty = 823$ MeV is much too large. This is a general problem of these simple mean field models and could be corrected by adding further meson interactions or potential terms to the Lagrangian.

The saturation value for the quark concentration is $x_s = 50\%$. This rather high value is reduced in finite nuclei due to the large surface of low nuclear matter density. A simple estimation with a Fermi distribution for the density
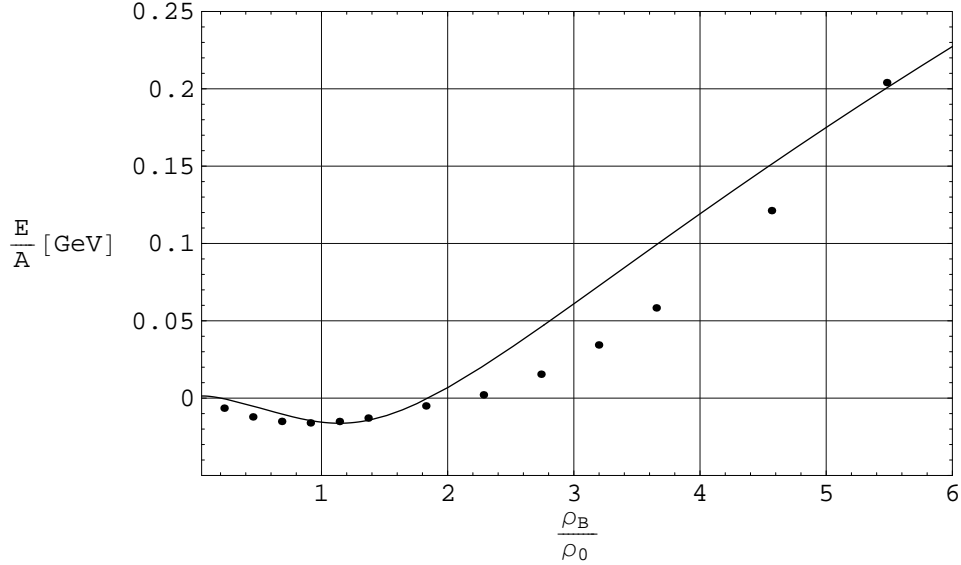
Figure 2: Nuclear matter equation of state

gives $\langle x \rangle \approx 40\%$ for ^{208}Pb and $\langle x \rangle < 30\%$ for ^{16}O. There are indeed experiments on the occupancies of lower shell model orbitals, which suggest that these orbitals are not entirely filled. Instead the Fermi surface is smeared out. This effect has been explained by a mixing of shell model orbitals, but in our model it is simply due to the coupling of different quark momenta to a baryonic three quark system. Such a composite baryon is not necessarily a nucleon but may be another excited baryonic state. Fig.3 shows the occupancies in lead computed with this model and looks almost the same as the result obtained under the inclusion of ground state correlations [8]. In the model described here, the "free" constituent quarks reduce the amount of quasi baryons below the Fermi momentum and enhance their number with momenta higher than k_{Fn}. The unchanged part of the Fermi surface is connected to the residue factor z, which can be measured. Because the 3s-orbital in lead is nearly exclusively located in the center of the nucleus, one can take in very good approximation the nuclear matter value of the quark concentration to compute the residue factor for this orbital. The result of $z = 0.67$ compares favorably to the value $z = 0.64 \pm 0.06$ from (e, e′p) experiments [9].
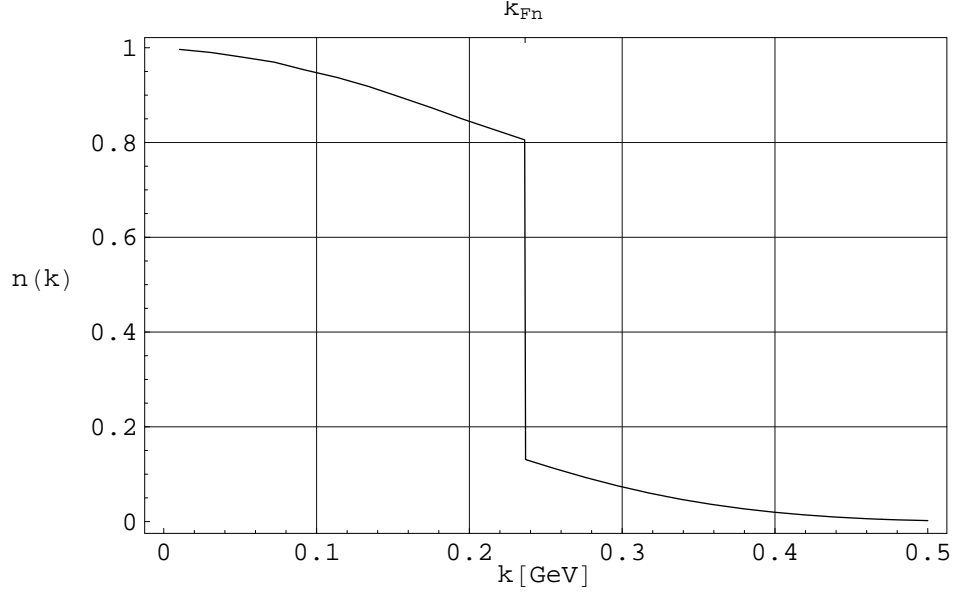
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Figure 3: Occupation numbers n(k) of quasi baryons in lead

4 Conclusion

Because of the repulsive ω-mean field at saturation $g_{vn}\bar{\omega}_s = 263$ MeV, the nucleons occupy the higher energy levels, whereas the constituent quarks have a strong occupancy in the deep lying levels. In such a way the overlap of constituent quarks and nucleons is restricted to the upper levels and would lead to a small correction of the energy density. Note that this overlap remains small at densities regarded here. The validity of our calculation is also limited by the credibility of the mean field approximation and the appearance of gluonic degrees of freedom at high density. We estimate the momentum scale for this region to be $k_{UV} \geq 1$ GeV cf. with our RG calculation [10]. Due to the nonabelian nature of QCD the three body gluon exchange forces are more important than the two body gluon exchanges. The three body forces lead to baryonic tripling whereas two body forces induce diquark formation [11]. The latter may be an intermediate stage on the way from high density to low density, but as shown in nature color neutralization will win ultimately at lower densities.

To summarize, we have constructed a very simplified model with constituent quarks and nucleons, which interpolates between low density nucleon matter and high density quark matter in a continuous way. As a result we have shown that our model qualitatively reproduces the nuclear matter EOS and provides a simple explanation for experimental measured residue factors.

References

[1] J.P. Blaizot, Phys. Rep. **64** (1980) 171

[2] J.D. Walecka, Theoretical Nuclear and Subnuclear Physics, Oxford University Press (1995)

[3] G.E. Brown and M. Rho, Phys. Rev. Lett. **66** (1991) 2720

[4] G.E. Brown, M. Buballa and M. Rho, Nucl. Phys. A **609** (1996) 519

[5] T. Kunihiro, J. Meyer, G. Papp and H.J. Pirner, to be published

[6] A. Akmal, V.R. Pandharipande, D.G. Ravenhall, Phys. Rev. C **58** (1998) 1804

[7] M. Jamion, C. Mahaux, Phys. Rev. C **40** (1989) 354

[8] R.W. Hasse, B.L. Friman and D. Berdichevsky, Phys. Lett. B **181** (1986) 5

[9] I. Sick and P.K.A. de Witt Huberts, Comm. Nucl. Part. Phys. **40** (1991) 177

[10] B.J. Schäfer and H.J. Pirner, hep-ph/9903003, subm. to Nucl. Phys. A

[11] M. Alford, K. Rajagopal and F. Wilczek, Phys. Lett. B **422** (1998) 247